

2.0 Chapter Introduction

In this chapter, you will learn to use cost-volume-profit analysis.

Assumptions. When you acquire supplies or services, you normally expect to pay a smaller price per unit as the purchase quantity increases. You expect contractors to have lower costs per unit as production quantity increases. This general expectation remains the same whether you are buying items specifically built for the Government or items that are mass-produced for a variety of commercial and Government customers. You can use cost-volume-profit analysis to analyze the natural relationship between cost, volume, and profit in pricing decisions. In cost-volume-profit analysis, you:

- Should consider only short-term operations. The short term may be defined as a period too short to permit facilities expansion or contraction or other changes that might affect overall pricing relationships.
- Assume that a straight line can reasonably be used in analysis. While actual price behavior may not follow a straight line, its use can closely approximate actual cost behavior in the short run.
 - If purchase volume moves outside the relevant range of the available data, the straight-line assumption and the accuracy of estimates become questionable.
 - If you know that product variable costs per unit are decreasing as quantity increases, consider using the log-linear improvement curve concept. Improvement curves are particularly useful in limited production situations where you can obtain cost/price information for all units sold.

Types of Cost. In the short run, costs can be of three general types:

- **Fixed Cost.** Total fixed costs remain constant as volume varies in the relevant range of production. Fixed cost per unit decreases as the cost is spread over an increasing number of units.

Examples include: Fire insurance, depreciation, facility rent, and property taxes.

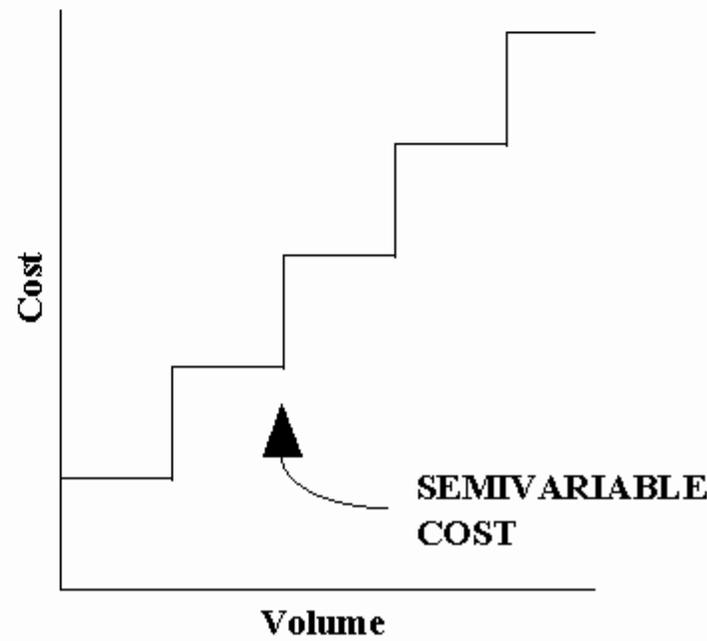
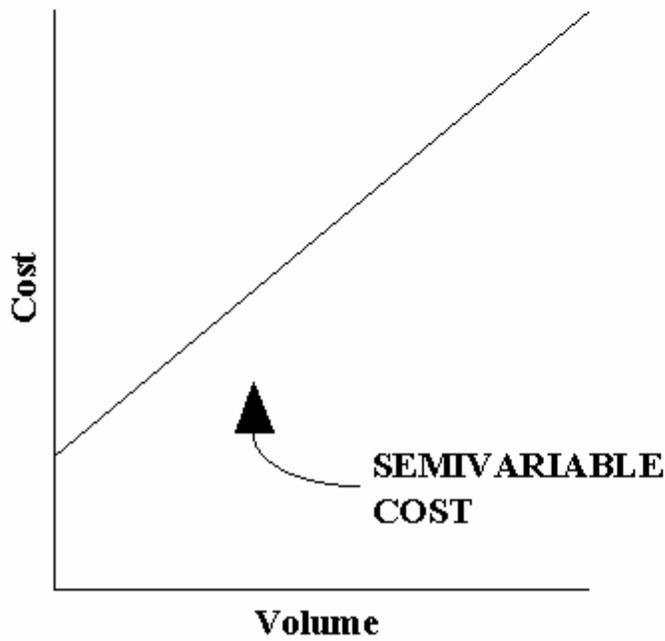
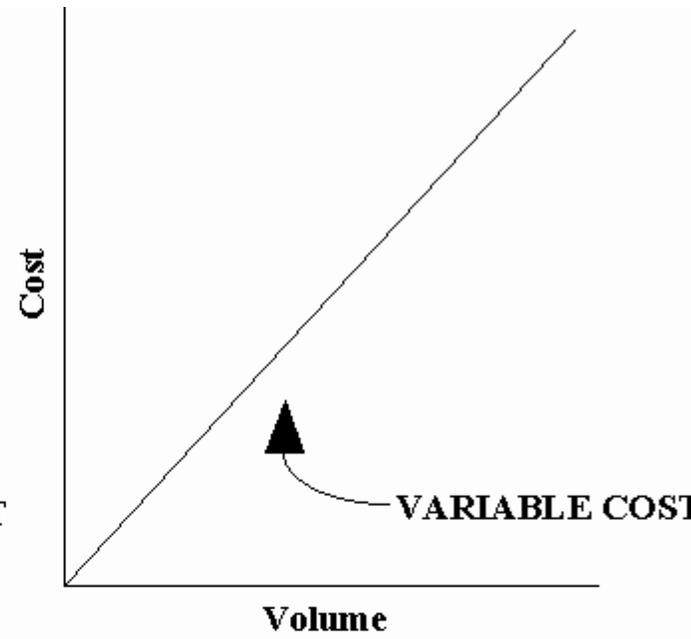
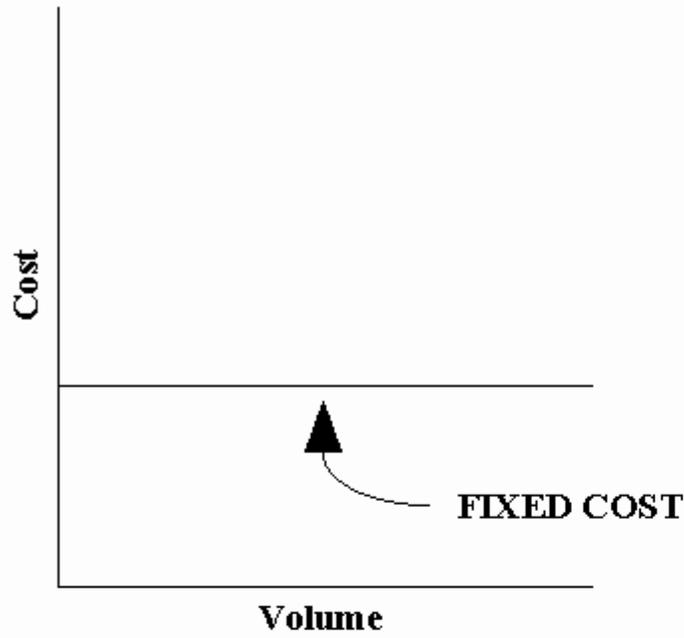
- **Variable Cost.** Variable cost per unit remains constant no matter how many units are made in the relevant range of production. Total variable cost increases as the number of units increases.

Examples include: Production material and labor. If no units are made, neither cost is necessary or incurred. However, each unit produced requires production material and labor.

- **Semi-variable Cost.** Semi-variable costs include both fixed and variable cost elements. Costs may increase in steps or increase relatively smoothly from a fixed base.

Examples include: Supervision and utilities, such as electricity, gas, and telephone. Supervision costs tend to increase in steps as a supervisor's span of control is reached. Utilities typically have a minimum service fee, with costs increasing relatively smoothly as more of the utility is used.

Graphic Depiction of Cost Behavior. The four graphs below illustrate the different types of cost behavior described above:



Profit. Profit is the difference between total cost and revenue. In cost-volume-profit analysis, a loss is

expressed as a negative profit. Breaking even, which is neither profit nor loss, is a profit of zero dollars.

2.1 Identifying Situations For Use

Situations for Use. Cost-volume-profit analysis is an estimating concept that can be used in a variety of pricing situations. You can use the cost-volume relationship for:

- **Evaluating item price in price analysis.** Cost-volume-profit analysis assumes that total cost is composed of fixed and variable elements. This assumption can be used to explain price changes as well as cost changes. As the volume being acquired increases unit costs decline. As unit costs decline, the vendor can reduce prices and same make the same profit per unit.
- **Evaluating direct costs in pricing new contracts.** Quantity differences will often affect direct costs -- particularly direct material cost. Direct material requirements often include a fixed component for development or production operation set-up. As that direct cost is spread over an increasing volume unit costs should decline.
- **Evaluating direct costs in pricing contract changes.** How will an increase in contract effort increase contract price? Some costs will increase while others will not. The concepts of cost-volume-profit analysis can be an invaluable aid in considering the effect of the change on contract price.
- **Evaluating indirect costs.** The principles of cost-volume-profit analysis can be used in indirect cost analysis. Many indirect costs are fixed or semi-variable. As overall volume increases, indirect cost rates typically decline because fixed costs are spread over an increasing production volume.

2.2 Analyzing The Cost-Volume Relationship

This section examines algebraic and graphic analysis of the cost-volume relationship.

- 2.2.1 - Algebraic Analysis

- 2.2.2 - Graphic Analysis

2.2.1 Algebraic Analysis

Key Assumption. The assumption of linear cost behavior permits use of straight-line graphs and simple linear algebra in cost-volume analysis.

Calculating Total Cost Algebraically. Total cost is a semi-variable cost—some costs are fixed, some costs are variable, and others are semi-variable. In analysis, the fixed component of a semi-variable cost can be treated like any other fixed cost. The variable component can be treated like any other variable cost. As a result, we can say that:

Total Cost = Fixed Cost + Variable Cost

Using symbols:

$$C = F + V$$

Where:

C = Total cost

F = Fixed cost

V = Variable cost

Total variable cost depends on two elements:

Variable Cost = Variable Cost per Unit x Volume Produced

Using symbols:

$$V = V_U(Q)$$

Where:

V_U = Variable cost per unit

Q = Quantity (volume) produced

Substituting this variable cost information into the basic total cost equation, we have the equation used in cost-volume analysis:

$$C = F + V_U (Q)$$

Example of Calculating Total Cost Algebraically. If you know that fixed costs are \$500, variable cost per unit is \$10, and the volume produced is 1,000 units, you can calculate the total cost of production.

$$\begin{aligned} C &= F + V_U (Q) \\ &= \$500 + \$10 (1,000) \\ &= \$500 + \$10,000 \\ &= \$10,500 \end{aligned}$$

Example of Calculating Variable Cost Algebraically. Given total cost and volume for two different levels of production, and using the straight-line assumption, you can calculate variable cost per unit.

Remember that:

- Fixed costs do NOT change no matter what the volume, as long as production remains within the relevant range of available cost information. Any change in total cost is the result of a change in total variable cost.
- Variable cost per unit does NOT change in the relevant range of production.

As a result, we can calculate variable cost per unit (V_U) using the following equation:

$$\begin{aligned} V_U &= \frac{\text{Change in Total Cost}}{\text{Change in Volume}} \\ &\text{or} \\ &= \frac{C_2 - C_1}{Q_2 - Q_1} \end{aligned}$$

Where:

C_1 = Total cost for Quantity 1

C_2 = Total cost for Quantity 2

Q_1 = Quantity 1

Q_2 = Quantity 2

You are analyzing an offeror's cost proposal. As part of the proposal the offeror shows that a supplier offered 5,000 units of a key part for \$60,000. The same quote offered 4,000 units for \$50,000. What is the apparent variable cost per unit?

$$\begin{aligned}V_U &= \frac{C_2 - C_1}{Q_2 - Q_1} \\ &= \frac{\$60,000 - \$50,000}{5,000 - 4,000} \\ &= \frac{\$10,000}{1,000} \\ &= \$10\end{aligned}$$

Example of Calculating Fixed Cost Algebraically. If you know total cost and variable cost per unit for any quantity, you can calculate fixed cost using the basic total cost equation.

You are analyzing an offeror's cost proposal. As part of the proposal the offeror shows that a supplier offered 5,000 units of a key part for \$60,000. The apparent variable cost is \$10 per unit. What is the apparent fixed cost?

$$\begin{aligned}C &= F + V_U(Q) \\ \$60,000 &= F + \$10(5,000) \\ \$60,000 &= F + \$50,000 \\ \$60,000 - \$50,000 &= F \\ \$10,000 &= F\end{aligned}$$

Developing an Estimating Equation. Now that you know that V_U is \$10 and F is \$10,000 you can substitute the values into the general total cost equation.

$$\begin{aligned}C &= F + V_U(Q) \\ &= \$10,000 + \$10(Q)\end{aligned}$$

You can use this equation to estimate the total cost of any volume in the relevant range between 4,000 and 5,000 units.

Using the Estimating Equation. Using the estimating equation for the relevant range, estimate the total cost of 4,400 units.

$$\begin{aligned} C &= \$10,000 + \$10(Q) \\ &= \$10,000 + \$10(4,400) \\ &= \$10,000 + \$44,000 \\ &= \$54,000 \end{aligned}$$

2.2.2 Graphic Analysis

Introduction to Graphic Analysis. When you only have two data points, you must generally assume a linear relationship. When you get more data, you can examine the data to determine if there is truly a linear relationship.

You should always graph the data before performing an algebraic analysis.

- Graphic analysis is the best way of developing an overall view of cost-volume relationship.
- Graphic analysis is useful in analyzing cost-volume relationships, particularly, when the cost and volume numbers involved are relatively small.
- Even when actual analysis is performed algebraically you can use graphs to demonstrate cost-volume analysis to others.

Steps of Graphic Analysis. There are four steps in using graph paper to analyze cost-volume relationships:

Step 1. Determine the scale that you will use. Volume is considered the independent variable and will be graphed on the horizontal axis. Cost is considered the dependent variable and will be graphed on the vertical axis. The scales on the two axes do not have to be the same. However, on each axis one block must represent the same amount of change as every other block of the same size on that axis. Each scale should be large enough to permit analysis, and small enough to permit the graphing of all available data and anticipated data estimates.

Step 2. Plot the available cost-volume data. Find the volume given for one of the data points on the horizontal axis. Draw an imaginary vertical line from that point. Find

the related cost on the vertical axis and draw an imaginary horizontal line from that point. The point where the two lines intersect represents the cost for the given volume. (If you do not feel comfortable with imaginary lines you may draw dotted lines to locate the intersection.) Repeat this step for each data point.

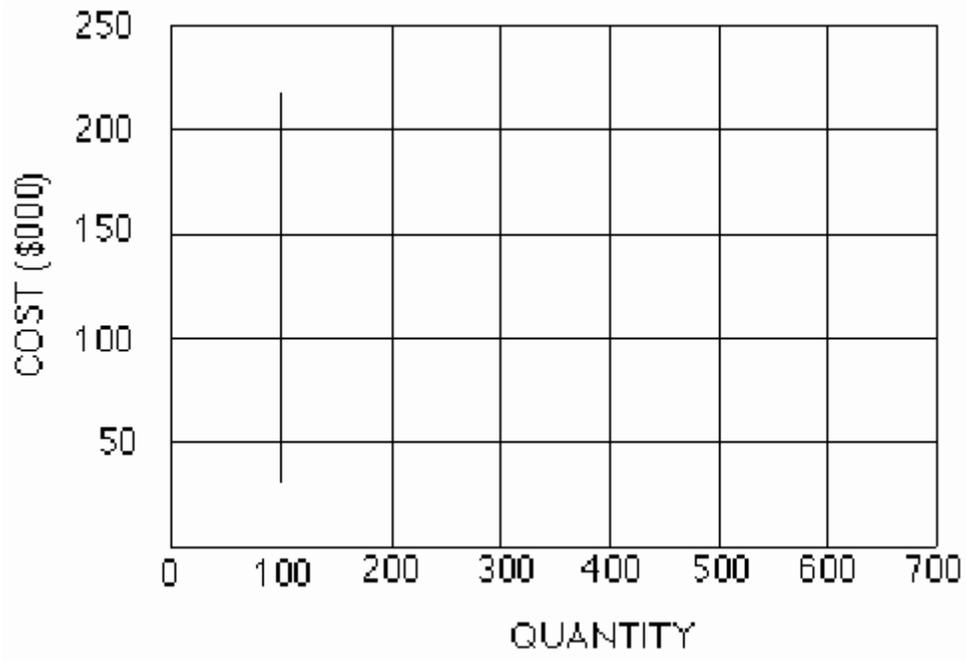
Step 3. Fit a straight line to the data. In this section of text, all data points will fall on a straight line. All that you have to do to fit a straight line is connect the data points. Most analysts use regression analysis to fit a straight line when all points do not fall on the line.

Step 4. Estimate the cost for a given volume. Draw an imaginary vertical line from the given volume to the point where it intersects the straight line that you fit to the data points. Then move horizontally until you intersect the vertical axis. That point is the graphic estimate of the cost for the given volume of the item.

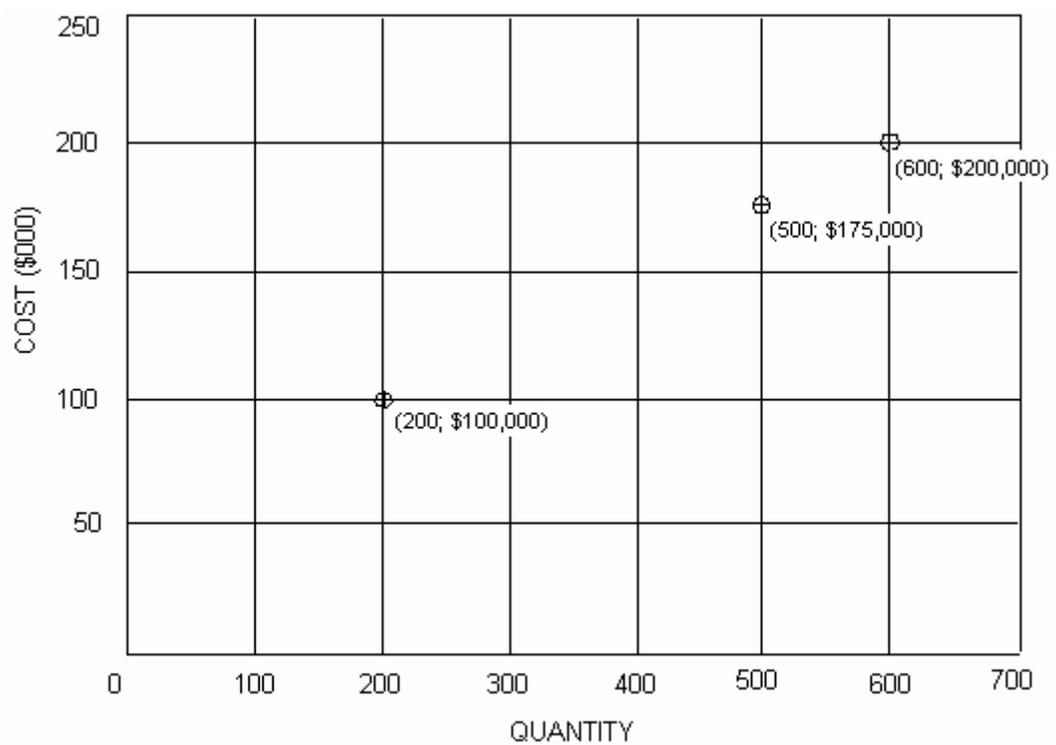
Example of Graphic Analysis. The four steps of cost-volume-profit analysis can be used to graph and analyze any cost-volume relationship. Assume that you have been asked to estimate the cost of 400 units given the following data:

Units	Cost
200	\$100,000
500	\$175,000
600	\$200,000

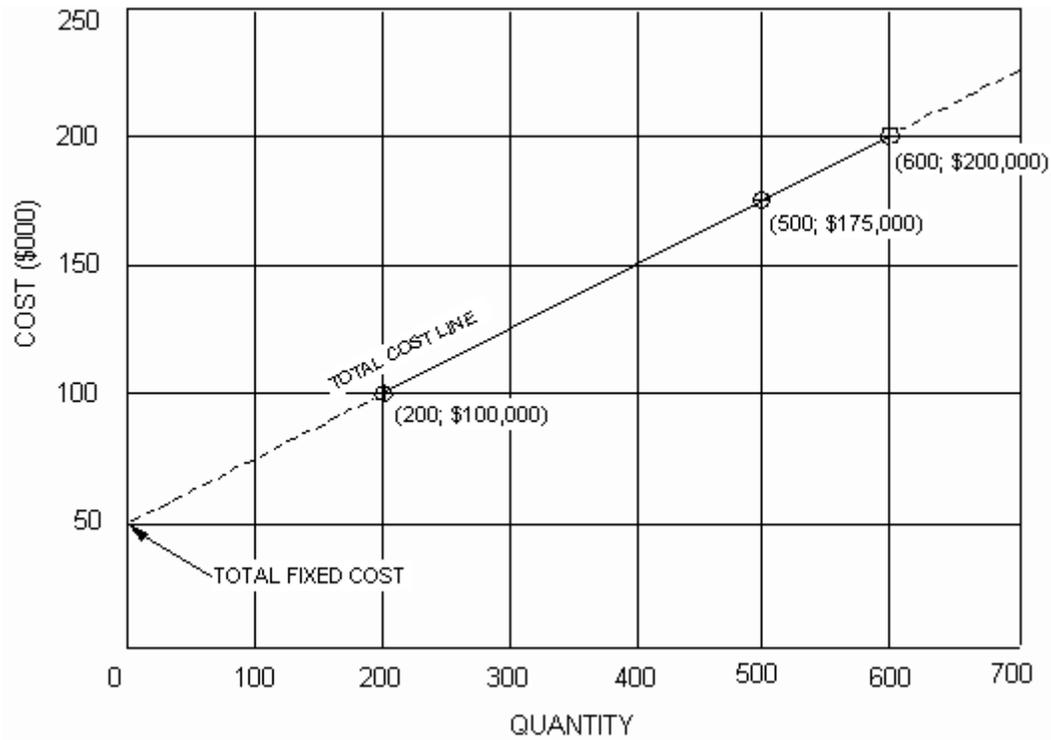
Step 1. Determine the scale that you will use.



Step 2. Plot the available cost-volume data.

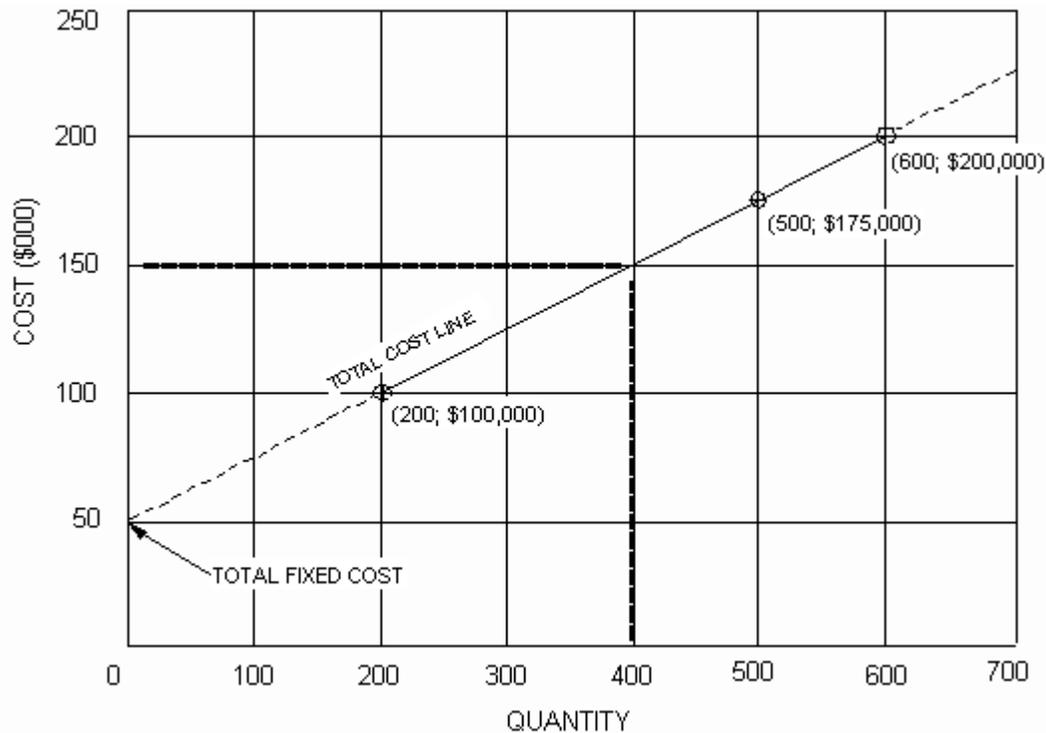


Step 3. Fit a straight line to the data.



Step 4. Estimate the cost for a given volume. From the graph, you can estimate that the total cost of 400 units will be \$150,000.

In addition you can also estimate fixed cost. The cost of making zero units, \$50,000, is the fixed cost for this set of data.



You can use any two points on the line to calculate the equation of the line. Follow the procedures in the section on Algebraic Analysis.

2.3 Analyzing The Price-Volume Relationship

Analysis Situations. In situations where you do not have offeror cost data, you can use the principles of the cost-volume relationship in price-volume analysis. Price-volume analysis is based on pricing information that is typically available to the Government negotiator.

Quantity Price Discounts. Unit prices normally decline as volume increases, primarily because fixed costs are being divided by an increasing number of units. Buyers see these price reductions with increasing volume in the form of quantity discounts.

Quantity discounts complicate the pricing decision, because a price that is reasonable for one volume may not be reasonable for a different volume.

Offers quote quantity discounts because their costs per unit declines as volume increases. As a result, even though offered prices include profit, you can use the cost-volume equation to estimate prices at different quantities.

Steps for Estimating a Quantity Price Discount. Estimating a quantity price discount is a 4-step process:

Step 1. Calculate the variable element.

Step 2. Calculate the fixed element using data from the available data points.

Step 3. Develop an estimating equation.

Step 4. Estimate the price for a given quantity

Example of Estimating a Quantity Price Discount. You know that the unit price for 100 units is \$3,000 and the unit price for 500 units is \$2,500. You are about to purchase 250 units. How can you use the available information to estimate the price for 250 units?

Approach this question just as though you were being asked to estimate cost given information for two different quantities. The only difference is that you use price data instead of cost data in the analysis.

Step 1. Calculate the variable element.

$$\begin{aligned}V_U &= \frac{C_2 - C_1}{Q_2 - Q_1} \\&= \frac{\$1,250,000 - \$300,000}{500 - 100} \\&= \frac{\$950,000}{400} \\&= \$2,375\end{aligned}$$

Step 2. Calculate the fixed element using data from the available data points.

$$\begin{aligned}C &= F + V_U(Q) \\ \$300,000 &= F + \$2,375(100) \\ \$300,000 - \$2,375(100) &= F \\ \$300,000 - \$237,500 &= F \\ \$62,500 &= F\end{aligned}$$

Step 3. Develop an estimating equation.

$$\begin{aligned} C &= F + V_U(Q) \\ &= \$62,500 + \$2,375(Q) \end{aligned}$$

Step 4. Estimate the price for 250 units.

$$\begin{aligned} C &= \$62,500 + \$2,375(Q) \\ &= \$62,500 + \$2,375(250) \\ &= \$62,500 + \$593,750 \\ &= \$656,250 \text{ total price and a } \$2,625 \text{ unit price} \end{aligned}$$

2.4 Analyzing The Cost-Volume-Profit Relationship

Until now, we have only looked at the cost-volume or price-volume relationship. Now, we are going to expand that relationship to consider the relationship between cost, volume, and profit.

Cost-Volume-Profit Equation. The revenue taken in by a firm is equal to cost plus profit. That can be written:

$$\text{Revenue} = \text{Total Cost} + \text{Profit}$$

We have already seen that total cost (C) is:

$$C = F + V_U(Q)$$

Using this information, we can rewrite the revenue equation as:

$$\text{Revenue} = F + V_U(Q) + \text{Profit}$$

In the cost-volume-profit equation, **profit can be positive, negative, or zero**. If profit is negative, we normally refer to it as a loss. If profit is zero, the firm is breaking even with no profit or loss. If we let P stand for profit, we can write the equation:

$$\text{Revenue} = F + V_U(Q) + P$$

Revenue is equal to selling price per unit (R_U) multiplied by volume.

$$\text{Revenue} = R_U(Q)$$

If we assume that the firm makes all the units that it sells, and sells all the units that it makes, we can complete the cost-volume-profit equation:

$$R_U(Q) = F + V_U(Q) + P$$

Application of the Cost-Volume-Profit Equation. This equation and limited knowledge of a contractor's cost structure can provide you with extremely valuable information on the effect purchase decisions can have on a firm's profitability.

Using the Cost-Volume-Profit Equation to Estimate Selling Price. Given the following product information, a firm prepared an offer for an indefinite quantity contract with the Government for a new product developed by the firm. There are no other customers for the product. In developing the offer unit price estimate (R_U), the firm used its estimated costs and its best estimate of the quantity that it would sell under the contract.

Fixed Cost = \$10,000

Variable Cost per Unit = \$20

Contract Minimum Quantity = 3,000 units

Contract Maximum Quantity = 6,000 units

Firm's Best Estimate of Quantity = 5,000 units

Target Profit = \$5,000

$$\begin{aligned} R_U(Q) &= F + V_U(Q) + P \\ R_U(5,000) &= \$10,000 + \$20(5,000) + \$5,000 \\ R_U(5,000) &= \$10,000 + \$100,000 + \$5,000 \\ R_U(5,000) &= \$115,000 \\ R_U &= \frac{\$115,000}{5,000} \\ R_U &= \$23.00 \end{aligned}$$

Using the Cost-Volume-Profit Equation to Estimate Profit. Managers of the firm wanted to know how profits would be

affected if it actually sold the maximum quantity (6000 units) at \$23.00 per unit.

$$\begin{aligned}R_U(Q) &= F + V_U(Q) + P \\ \$23(6,000) &= \$10,000 + \$20(6,000) + P \\ \$138,000 &= \$10,000 + \$120,000 + P \\ \$138,000 &= \$130,000 + P \\ \$138,000 - \$130,000 &= P \\ \$8,000 &= P\end{aligned}$$

If the firm sells 6,000 units at \$23.00 per unit, profit will be \$8,000. That is a \$3,000 increase from the original \$5,000 target profit, or an increase of 60 percent. Note that the firm's profit would increase solely because sales were higher than estimated.

Managers were even more concerned about how profits would be affected if they only sold the minimum quantity (4000 units) at \$23.00 per unit.

$$\begin{aligned}R_U(Q) &= F + V_U(Q) + P \\ \$23(4,000) &= \$10,000 + \$20(4,000) + p \\ \$92,000 &= \$10,000 + \$80,000 + P \\ \$92,000 &= \$90,000 + P \\ \$2,000 &= P\end{aligned}$$

If the firm sells 4,000 at \$23.00 per unit, profit will be \$2,000. That is \$3,000 less than the original \$5,000 target profit. Note that the firm's profit would decrease solely because sales were lower than estimated.

Using the Cost-Volume-Profit Equation to Estimate Break-Even Sales. In a final effort to analyze the risk to the firm under the proposed indefinite deliver contract, managers wanted to know the level of sales that would be required for the firm to break even (zero profit).

$$\begin{aligned}
R_U(Q) &= F + V_U(Q) + P \\
\$23(Q) &= \$10,000 + \$20(Q) + 0 \\
(\$23 - \$20)(Q) &= \$10,000 \\
\$3(Q) &= \$10,000 \\
Q &= \frac{\$10,000}{3} \\
Q &= 3,333.33 \text{ units}
\end{aligned}$$

The calculations show that the firm would break even at 3,333.33 units. Assuming that the firm could not sell .33 units, the firm must sell 3,334 units to assure that all costs are covered. Selling 3,333 units would still result in a \$.10 loss.

Contribution Income. The difference between revenue and variable cost is contribution income (I). The term contribution income comes from the contribution made to covering fixed costs and profit. If contribution income is positive, increasing sales will increase profits or reduce losses. If contribution income is negative, increasing sales will reduce profits or create greater losses.

Contribution Income = Revenue - Variable Cost

Using symbols:

$$\begin{aligned}
I &= R_U(Q) - V_U(Q) \text{ or} \\
I &= (R_U - V_U)(Q)
\end{aligned}$$

Knowledge of a contractor's cost structure and contribution income can be valuable in analysis of proposed costs.

Contribution Income Example. In evaluating an offeror's proposal for 500 units at \$900 each, your analysis reveals the following cost structure:

$$\begin{aligned}
\text{Fixed Cost} &= \$100,000 \\
\text{Variable Cost per Unit} &= \$1,000
\end{aligned}$$

How would this affect your analysis of contract risk?

$$\begin{aligned}
I &= (R_U - V_U)(Q) \\
&= (\$900 - \$1,000)(500) \\
&= -\$100(500) \\
&= -\$50,000
\end{aligned}$$

The contribution income from the sale is a negative \$50,000. The firm would be substantially worse off for having made the sale. Unless the firm can offer a positive rationale for such a pricing decision, you must consider pricing as an important factor as you analyze the risk of contract performance.

2.5 Identifying Issues And Concerns

Questions to Consider in Analysis. As you perform price/cost analysis, consider the issues and concerns identified in this section, whenever you use cost-volume-profit analysis concepts.

- ***Has the contractor's cost structure changed substantially?***

Application of cost-volume-profit analysis assumes that the period covered by the analysis is too short to permit facilities expansion or contraction or other changes that might affect overall pricing relationships. If the contractor has substantially changed its cost structure, your ability to use cost-volume-profit analysis may be limited. Examples of possible changes include:

- Downsizing to reduce fixed costs; and
- Increased investment in automated equipment to reduce variable costs of labor and material.

- ***Is the straight-line assumption reasonable?***

The cost-volume-profit relationship is not usually a straight-line relationship. Instead, it is a curvilinear relationship. A straight-line analysis works as long as the straight line is a good approximation of the cost-volume-profit relationship. Most computer programs designed to fit a straight-line to a set of data provide measures of how well the line fits the data. For example, a regression program will usually provide the coefficient of determination (r^2).

- ***Are current volume estimates within the relevant range of available data?***

If the current business volume is substantially higher or lower than the volumes used to develop the cost-volume-profit equation, the results may be quite unreliable. The contractor should be expected to change the way it does business and its cost structure if volume increases or decreases substantially.